Performance of Quasi-Newton Method for Estimation of Relative Permittivity in 1-D Inverse Scattering Problem

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The microwave tomography is one of effective technique for estimating the material distribution. When the objective function is defined, some optimization techniques can be introduced into the microwave tomography. The line search strategy is usually used for updating solution in gradient-based method. However, this strategy impacts strongly convergence property and elapsed time. On the other hand, when the quasi-Newton (QN) method is implemented, the cost for line search would be successfully reduced. This paper investigates the performance of the QN method in 1-D inverse scattering problem. Furthermore, to reduce the computational cost for line search, the linear approximation of derivative of objective function is applied to determination of the step size.

***Index Terms*—FDTD method, inverse problems, optimization methods, tomography.**

I. Introduction

THE microwave tomography is a hopeful technique for nondestructive testing, medical imaging, geophysical exploration, etc. The distribution of permittivity ε, magnetic permeability μ, and conductivity σ is reconstructed by making use of the measurement of scattering waves.

When the objective function is defined, some optimization techniques can be introduced into microwave tomography. The reconstruction algorithm based on a gradient-based method [1] has been developed. The line search strategy is usually carried out in order to accelerate convergence of objective function. Because this strategy affects the elapsed time, the cost reduction of line search is desired for the large scale problem. On the other hand, the quasi-Newton (QN) method [2] is widely implemented in optimization problems. Because the converged solution is usually obtained by QN method with step size α = 1.0, the line-search cost might be successfully reduced. However, the performance of QN method has not been demonstrated in detail on inverse scattering problem.

This paper investigates the performance of QN method for estimating relative permittivity in 1-D inverse scattering problem. The convergence property using QN method is compared with conjugate gradient (CG) method. Furthermore, to accelerate the convergence and computational speed, the line search based on linear approximation of derivative of objective function [3] is newly introduced into QN method.

II. Optimization Technique

A. Quasi Newton (QN) Method

Fig. 1 shows the procedure of QN method. Here, $F(\boldsymbol{x}_k)$ denotes the objective function. The relative permittivity ε_r in target domain is stored on the vector $\boldsymbol{x}_k$. H_k represents $n \times n$ square matrix, where n is the number of unknowns. In step 2, to determine the search direction $\boldsymbol{d}_k$, the design sensitivity $\nabla F(\boldsymbol{x}_k)$ is required. To efficiently calculate sensitivity, time-domain adjoint-variable method (AVM) [1], [4] is performed in this paper. In step 3, a step size α is determined so that the objective function can decrease monotonously. One of the line search technique is the golden section (GS) technique. However, because the evaluation of objective function is repeated until convergence, this approach might be inadequate from the viewpoint of elapsed time.

Step 1 (Initialization) Let $\boldsymbol{x}_0$ be an initial guess. Set $H_0 = I$, $k = 0$.
Step 2 (Calculation of search direction) $\boldsymbol{d}_k = -H_k \nabla F(\boldsymbol{x}_k)$
Step 3 (Determination of step size) $\alpha_k = \min_{\alpha} F(\boldsymbol{x}_k + \alpha \boldsymbol{d}_k)$
Step 4 (Update of solution) $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha_k \boldsymbol{d}_k$
Step 5 (Convergence check)
When $|\delta \boldsymbol{x}| < \varepsilon$ is satisfied, QN iteration is terminated; otherwise go to Step 6.
Step 6 (Update of vectors) $\boldsymbol{s}_k = \boldsymbol{x}_{k+1} - \boldsymbol{x}_k$, $\boldsymbol{y}_k = \nabla F(\boldsymbol{x}_{k+1}) - \nabla F(\boldsymbol{x}_k)$.
Step 7 (Update of matrix) $H_{k+1} = H_k - \dfrac{H_k \boldsymbol{y}_k \boldsymbol{s}_k^T + \boldsymbol{s}_k (H_k \boldsymbol{y}_k)^T}{\boldsymbol{s}_k^T \boldsymbol{y}_k} + \left(1 + \dfrac{\boldsymbol{y}_k^T H_k \boldsymbol{y}_k}{\boldsymbol{s}_k^T \boldsymbol{y}_k}\right) \dfrac{\boldsymbol{s}_k \boldsymbol{s}_k^T}{\boldsymbol{s}_k^T \boldsymbol{y}_k}$
Step 8 (Update of iteration index) $k = k+1$, return to Step 2.

Fig. 1. Algorithm of QN method.

B. Line Search Technique based on Linear Approximation of Derivative of Objective Function

To efficiently determine the step size α, the line search strategy based on linear approximation of $\partial F/\partial \alpha$ [3] is newly applied to QN method. Fig. 2 shows the procedure of linear approximation of $\partial F/\partial \alpha$. The one-dimensional nonlinear equation $\partial F/\partial \alpha = 0$ is approximately solved by the two-point linear interpolation as shown in Fig. 2. Here, $\partial F/\partial \alpha$ is evaluated as follows:

$$\frac{\partial F^{(k+1)}}{\partial \alpha} = \left(\frac{\partial F(\boldsymbol{x}_{k+1})}{\partial \boldsymbol{x}_{k+1}} \right)^T \frac{\partial \boldsymbol{x}^{(k+1)}}{\partial \alpha} = \nabla F(\boldsymbol{x}_{k+1})^T \boldsymbol{d}_k \,. \tag{1}$$

Although the sensitivity analysis is newly added, the cost of this approach is lower than that of GS method. In this paper, α_1 and α_2 is set to 0.5 and 1.5, respectively.

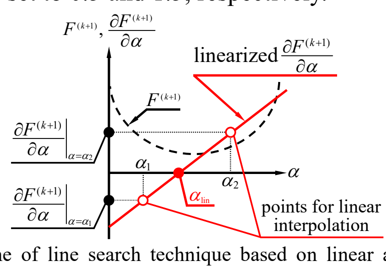

Fig. 2. Outline of line search technique based on linear approximation of $\partial F/\partial \alpha$.

On the other hand, because H_0 is set to the unit matrix in this paper, $\boldsymbol{d}_0$ is identical to steepest descent direction. Consequently, the line search based on Fig. 2 might be unsuitable for determining α_0. Therefore, the determination of step size is performed with Taylor expansion of objective function as follows:

$$F^{(1)}\left(\boldsymbol{x}_0-\alpha_0\frac{\partial F^{(0)}}{\partial \boldsymbol{x}_0}\right)\approx F^{(0)}-\alpha_0\left(\frac{\partial F^{(0)}}{\partial \boldsymbol{x}_0}\right)^T\frac{\partial F^{(0)}}{\partial \boldsymbol{x}_0}=0\,. \qquad (2)$$

As a result, α_0 can be obtained as:

$$\alpha_0 = F^{(0)}/\|\nabla F(\boldsymbol{x}_0)\|^2\,. \qquad (3)$$

In this paper, initial line search is performed by solving (3), and α_k ($k > 1$) is determined by the procedure shown in Fig. 2.

III. Numerical Result

Fig. 3 shows the 1-D analyzed model. Here, L (= 450 mm) is the maximum of x. The value shown in parentheses denotes the true value of relative permittivity. The objective function is formulated as follows:

$$F(\varepsilon_r)=\frac{\int_0^{T_{\max}}\{\boldsymbol{E}_m(t)-\boldsymbol{E}(\varepsilon_r,t)\}^2\,\mathrm{d}t}{\int_0^{T_{\max}}\{\boldsymbol{E}_m(t)\}^2\,\mathrm{d}t}, \qquad (4)$$

where $T_{\max}$ is the time duration of the measurement, $\boldsymbol{E}_m(t)$ is the measured electric field, and $\boldsymbol{E}(\varepsilon_r, t)$ is the electric field derived from the finite-difference time-domain (FDTD) method. Each layer is discretized by 10 cells, and all layers is assumed to a lossless material. A raised cosine pulse [5] is adopted as wave source. The search space of relative permittivity is from 1 to 12, and all initial ε_r is set to 1. When the maximum correction of relative permittivity $|\delta\varepsilon_r|_{\max}$ is less than 10^{-3}, the estimation process is terminated. The convergence criterion for golden section (GS) strategy is set to 10^{-8}.

Fig. 4 shows the convergence of objective function. In the case without line search (α = 1.0), the convergence speed of QN method is superior to that of CG method. On the other hand, when the GS technique is applied to both optimization methods, it can be seen that the convergence characteristic is improved in comparison with Fig. 4 (a). In this case, the convergence of CG method with GS is faster than that of QN method with GS. The QN method with developed line search also has the ability to improve the convergence as shown in Fig. 4 (b). Table I shows the performance of optimization methods. k_{opt} is the number of optimization step. N_{FDTD} and N_{AVM} denote the total required number of FDTD and AVM in line search, respectively. In the line search based on Fig. 2, because α can be determined by the two-point linear interpolation, the cost for developed line search is smaller than that for GS. Therefore, the elapsed time using QN with developed line search is the shortest among all methods.

Fig. 5 shows the characteristic of objective function and $\partial F/\partial\alpha$. In Fig. 6 (a), the step size closer to exact value α_{ex} is obtained. At the 9th iteration, the discrepancy between linearized $\partial F/\partial\alpha$ and exact $\partial F/\partial\alpha$ is minor so that the impact of higher order term in Taylor expansion becomes small with the approach to the true value. Fig. 6 shows the behavior of α during optimization process. It can be seen that α approaches to 1.0 in later QN iteration. In the full paper, the other numerical results will be illustrated.

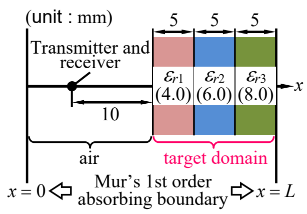

Fig. 3. Laminated model for estimating relative permittivity.

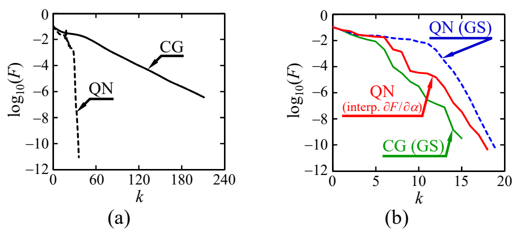

Fig. 4. Convergence property of objective function. (a) without line search (α = 1.0). (b) with line search.

TABLE I
RESULTANT PERFORMANCE OF OPTIMIZATION METHODS

optimization method	line search technique	k_{opt}	N_{FDTD}	N_{AVM}	elapsed time [s]
CG	GS	15	851	—	0.22
QN	GS	19	821	—	0.19
	interp. $\partial F/\partial\alpha$	18	34	34	0.04

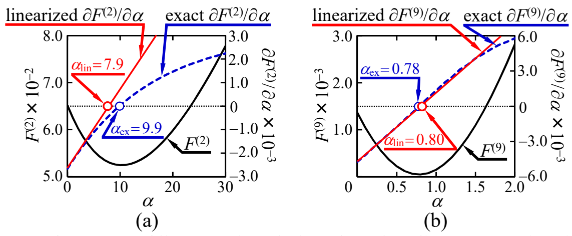

Fig. 5. Performance of line search technique based on linear approximation of $\partial F/\partial\alpha$ in QN method. (a) 2nd iteration. (b) 9th iteration.

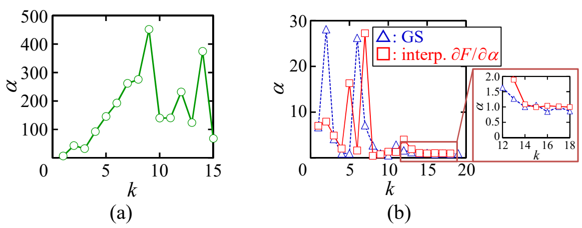

Fig. 6. Changes of step size. (a) CG method with GS. (b) QN method with line search techniques.

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